

# Charmonium production from the hadronic phase

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## Abstract

Charmonium production from the hadron gas formed in ultra-relativistic heavy-ion collisions is studied. Using the  $J/\psi$ -hadron absorption cross section determined from the nonperturbative quark-exchange model, which has a peak value similar to that used in the comover model for  $J/\psi$  suppression and a thermally averaged value consistent from that extracted from  $J/\psi$  data in heavy-ion collisions, we find that  $J/\psi$  production from the hadron gas is negligible in heavy-ion collisions at RHIC energies but is important at LHC energies as a result of the large number of charm mesons produced at higher energy collisions. The number of  $J/\psi$  produced from these secondary collisions at LHC may be comparable to that of primary  $J/\psi$ 's, which are expected to be dissociated in the quark-gluon plasma created in the collisions, leading thus to a possible absence of  $J/\psi$  suppression. Similar results are obtained for  $\psi'$  production in ultra-relativistic heavy-ion collisions.

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One of the signals proposed for identifying the existence of a quark-gluon plasma in ultra-relativistic heavy-ion collisions is the suppression of  $J/\psi$  production compared to that expected from the superposition of nucleon-nucleon collisions [1]. According to Matsui and Satz, if a quark-gluon plasma is created in heavy-ion collisions,  $J/\psi$ 's produced from initial nucleon-nucleon interactions will dissociate as a result of Debye screening and vanishing string tension between  $c$  and  $\bar{c}$ . Recent experimental results from heavy-ion collisions at CERN SPS energies [2–4] have indeed shown a reduction of  $J/\psi$  production. Part of the effects can be attributed to absorption by nucleons as it is more likely that  $J/\psi$  is first produced as a pre-resonance in a color octet state and thus has a larger interaction cross section with nucleon [5–8]. Such an effect is needed to account for the observed  $J/\psi$  suppression in proton-nucleus collisions [9], where one does not expect the formation of a quark-gluon plasma. Also,  $J/\psi$  absorption by comovers, which are most pions produced in the collisions, has been suggested [10–13]. In collisions of light ions such as S+U at 200 GeV/nucleon, these absorption mechanisms are sufficient to explain the experimental data. However, for central collisions of heavy nuclei such as Pb+Pb at 160 GeV/nucleon, the anomalous large suppression of  $J/\psi$  observed in the experiments has led to the suggestions that a quark-gluon plasma is formed in these collisions, and the  $J/\psi$  yield is reduced due to dissociation either by Debye screening [14–16] or collisions with gluons [17]. If the quark-gluon plasma has already been formed in heavy-ion collisions at SPS energies, it is almost certain that it will also be formed in heavy-ion collisions at the Relativistic Heavy-Ion Collider (RHIC) at the Brookhaven National Laboratory and the Large Hadron Collider at CERN (LHC), where energies are much higher. Therefore,  $J/\psi$  suppression at RHIC and LHC is considered as one of the most prominent signals for the quark-gluon plasma.

But will  $J/\psi$  production at RHIC and LHC be really reduced? This depends on whether  $J/\psi$  can be regenerated from the hadronic matter after the phase transition of the quark-gluon plasma. Such an effect has been shown to be important for hadrons made of strange quarks [18]. In hadronic matter,  $J/\psi$  can be produced from the interactions of charm mesons in reactions such as  $D\bar{D}^*, D^*\bar{D}, D^*\bar{D}^* \rightarrow J/\psi\pi$ . These reactions are apparently unimportant at SPS energies but may become significant at RHIC and LHC since charm production in nucleon-nucleon interaction increases with center-of-mass energy while the  $J/\psi$  to  $c\bar{c}$  ratio remains essentially at a constant value of  $\sim 2.5 \times 10^{-2}$  [19]. Furthermore, the reaction is exothermic as  $m_D + m_{\bar{D}^*} \sim 3.73 \text{ GeV}/c^2$  and  $m_{D^*} + m_{\bar{D}^*} \sim 3.87 \text{ GeV}/c^2$  while  $m_{J/\psi} + m_\pi \sim 3.15 \text{ GeV}/c^2$ , it is thus more likely that  $J/\psi$ 's produced from the hadronic matter will survive and be detected in experiments. If this is the case, then the final number of  $J/\psi$  may be comparable to or even larger than that of primary  $J/\psi$ 's which are dissociated in the quark-gluon plasma, leading instead to an absence of  $J/\psi$  suppression or even an enhanced  $J/\psi$  production. In the following, we shall make an estimate of  $J/\psi$  production from a pion gas formed in heavy-ion collisions at RHIC and LHC energies.

To model heavy-ion collisions at such high energies, we use the results from the HIJING calculation [20], which takes into account parton production from semihard scatterings. It shows that at an initial proper time  $\tau_0$  a thermally equilibrated although chemically non-equilibrated quark-gluon plasma of temperature  $T_0$  is formed. For Au+Au collisions, HIJING predicts that  $\tau_0 \sim 0.7$  and  $0.5 \text{ fm}/c$ ,  $T_0 \sim 0.57$  and  $0.83 \text{ GeV}$  at RHIC and LHC energies, respectively. The quark-gluon plasma then cools due to expansion and production of additional partons. It reaches the critical temperature  $T_c \sim 200 \text{ MeV}$  at about  $\tau_c \sim 3$

fm/c at RHIC and 6 fm/c at LHC, when the quark-gluon plasma starts to make a transition to a hadron gas, consisting mostly of pions. Since neither the partons have reached chemical equilibrium nor the order of phase transition is definitely known, the time for the system to remain at  $T_c$  can not be properly determined. We shall assume that the proper time at which the quark-gluon plasma is completely converted to a pion gas is  $\tau_h \sim 2\tau_c$  and will study how the results depend on the value of  $\tau_h$ . Since transverse expansion is not expected to be appreciable in this early phase, the initial volume of the hadronic matter is simply  $V_h \sim \pi R_0^2 \tau_h$  if one uses the boost-invariant model of Bjorken [21]. In the above,  $R_0$  is the radius of the colliding nuclei. Assuming that the hadron gas expands isentropically, its temperature then decreases according to the inverse of the cubic root of volume. To include the transverse expansion of the hadron gas, we introduce an acceleration  $a$ . Then its transverse radius increases with time according to  $R(\tau) = R_0 + a(\tau - \tau_h)^2/2$  until the velocity reaches the velocity of light, when the transverse radius increases linearly with time. From entropy conservation, the temperature of the hadron gas then decreases as  $T(\tau) = (\tau_h/\tau)^{1/3}(R_0/R(\tau))^{2/3}T_c$ . Final hadron yields and spectra are determined at freeze out with a temperature  $T_f \sim 120$  MeV. We note that for pions in equilibrium the density is approximately given by  $n_\pi \sim 0.25(T/197\text{MeV})^3 \text{ fm}^{-3}$ .

The time evolution of the temperature and volume of the hadron gas for Au+Au collisions at both RHIC and LHC is shown in Fig. 1 for  $a = 0.1 \text{ c}^2/\text{fm}$ . The transverse expansion velocity at freeze out is about  $0.75c$  and  $0.9c$  at RHIC and LHC, respectively. The initial number of charm quarks produced in these collisions can be estimated from their production cross section  $\sigma_{c\bar{c}}^{NN}$  in nucleon-nucleon interaction using [22]

$$N_{c\bar{c}}^{\text{AB}} \approx AB\sigma_{c\bar{c}}^{NN}/\sigma_{\text{in}}^{\text{AB}}, \quad (1)$$

where  $\sigma_{\text{in}}^{\text{AB}}$  is the nucleus-nucleus (A+B) inelastic cross section. For central collisions,  $\sigma_{\text{in}}^{\text{AB}}$  is approximately the nuclear geometrical cross section. The cross section  $\sigma_{c\bar{c}}^{NN}$  has been evaluated using the perturbative QCD [20], and it has a value 0.16 mb and 5.75 mb for nucleon-nucleon interaction at RHIC and LHC energies, respectively. For Au+Au collisions, the initial number of  $c\bar{c}$  (i.e.,  $D\bar{D}$ ) pairs, is thus about 3.8 at RHIC and 136 at LHC. To determine the initial  $J/\psi$  number, we assume that the same  $c\bar{c}$  to  $J/\psi$  ratio, about 40, observed in nucleon-nucleon interaction at center-of-mass energy below 40 GeV [19] holds at RHIC and LHC energies. In this case, the primary  $J/\psi$  number is about 0.095 and 3.4 at RHIC and LHC, respectively. Since a quark-gluon plasma is almost certain to be created in heavy-ion collisions at RHIC and LHC, both  $J/\psi$ 's and  $D$ 's are expected to be dissociated due to Debye screening. As shown in Ref. [20], thermal charm production from the quark-gluon plasma is small, it is thus reasonable to assume that the  $c$  and  $\bar{c}$  quark numbers remain constant during the quark-gluon phase. After phase transition to a hadron gas, these charm quarks are more likely to form  $D$  and  $\bar{D}$  mesons and their excited states due to the large abundance of light quarks in the plasma. Conservation of charm then requires that the number of charm mesons at the beginning of the hadron phase is the same as the initial one.

The  $J/\psi$  production cross section from  $D\bar{D}$  interaction,  $\sigma_{D\bar{D} \rightarrow J/\psi\pi}$ , can be related to the absorption cross section  $\sigma_{J/\psi\pi \rightarrow D\bar{D}}$  via the detailed balance relation

$$\sigma_{D\bar{D} \rightarrow J/\psi\pi} = d(k_{J/\psi\pi}/k_{D\bar{D}})^2 \sigma_{J/\psi\pi \rightarrow D\bar{D}}, \quad (2)$$

where the factor  $d$  is  $3/4$  for  $D\bar{D}^*$  and  $D^*\bar{D}$  annihilation, and  $1/4$  for  $D^*\bar{D}^*$  annihilation;  $k_{J/\psi\pi}$  and  $k_{D\bar{D}}$  are, respectively, the relative momenta of  $J/\psi\pi$  and  $D\bar{D}$ . The magnitude

of  $J/\psi$  absorption cross section by pion,  $\sigma_{J/\psi\pi\rightarrow D\bar{D}}$ , is still under debate. In the comover model for  $J/\psi$  suppression in heavy-ion collisions at SPS energies, it is taken to be about 3 mb. Both perturbative QCD [6] and effective Lagrangian [23] calculations give a value which is about a factor of 10 smaller. On the other hand, studies including nonperturbative effects via quark-exchange model [24] give a peak value of about 6 mb. As shown below, the thermally averaged effective cross section that takes into account the threshold effect is much smaller and is consistent with that extracted in Ref. [11] from  $J/\psi$  suppression data in heavy-ion collisions. In the following, we shall use the parameterized values from Ref. [24], i.e.,

$$\sigma_{J/\psi\pi\rightarrow D\bar{D}}(s) \approx \sigma_0 \left(1 - \left(\frac{s_0}{s}\right)^2\right) \exp[-a(\sqrt{s} - \sqrt{s_1})] \theta(s - s_0), \quad (3)$$

where  $s$  is the square of the  $J/\psi\pi$  center-of-mass energy and  $s_0$  is the threshold energy. For the final states  $D\bar{D}^*$  and  $\bar{D}D^*$ , the parameters are  $\sigma_0 = 2.5$  mb,  $s_0 = 15.05$  GeV<sup>2</sup>,  $s_1 = 17.6$  GeV<sup>2</sup>, and  $a = 11$  GeV<sup>-1</sup>; while for the final state  $D^*\bar{D}^*$ , they are  $\sigma_0 = 3.4$  mb,  $s_0 = 16.2$  GeV<sup>2</sup>,  $s_1 = 19.0$  GeV<sup>2</sup>, and  $a = 11$  GeV<sup>-1</sup>.

The quantity needed for estimating  $J/\psi$  production from  $D\bar{D}$  interaction in a hadron gas is the thermal average of  $\sigma_{D\bar{D}\rightarrow J/\psi\pi}v$ , where  $v$  is the relative velocity between  $D$  and  $\bar{D}$ , i.e.,

$$\begin{aligned} \langle \sigma_{D\bar{D}\rightarrow J/\psi\pi}v \rangle &= [4(m_{D_1}/T)^2(m_{D_2}/T)^2 K_2(m_{D_1}/T) K_2(m_{D_2}/T)]^{-1} \int_{(m_{D_1}+m_{D_2})/T}^{\infty} dz \\ &\cdot [z^2 - (m_{D_1}/T + m_{D_2}/T)^2][z^2 - (m_{D_1}/T - m_{D_2}/T)^2] \\ &\cdot K_1(z) \sigma_{D\bar{D}\rightarrow J/\psi\pi}. \end{aligned} \quad (4)$$

In the above,  $m_{D_1}$  and  $m_{D_2}$  are the masses of the two charm mesons;  $K_1$  and  $K_2$  are, respectively, the modified Bessel function of the first and second kind. The above quantity has been evaluated separately for  $D\bar{D}^*$ ,  $D^*\bar{D}$ , and  $D^*\bar{D}^*$  annihilation. In Fig. 2, we show the temperature dependence of  $\langle \sigma_{D\bar{D}^*}v \rangle$ , defined by

$$\langle \sigma_{D\bar{D}^*}v \rangle = \langle \sigma_{D\bar{D}^*\rightarrow J/\psi\pi}v \rangle + \langle \sigma_{D^*\bar{D}\rightarrow J/\psi\pi}v \rangle, \quad (5)$$

and  $\langle \sigma_{D^*\bar{D}^*}v \rangle = \langle \sigma_{D^*\bar{D}^*\rightarrow J/\psi\pi}v \rangle$ . Similarly, one evaluates the thermal average  $\langle \sigma_{J/\psi\pi\rightarrow D\bar{D}}v \rangle$  for the production of  $D\bar{D}^*$ ,  $D^*\bar{D}$ , and  $D^*\bar{D}^*$ , and defines  $\langle \sigma_{J/\psi\pi}v \rangle$  as their sum. The temperature dependence of  $\langle \sigma_{J/\psi\pi}v \rangle$  is also shown in Fig. 2. We see that  $\langle \sigma_{D^*\bar{D}^*}v \rangle$  is about a factor of three smaller than  $\langle \sigma_{D\bar{D}}v \rangle$  due to a smaller cross section. Furthermore,  $\langle \sigma_{D\bar{D}}v \rangle$  is larger than  $\langle \sigma_{J/\psi\pi}v \rangle$  at all temperatures. The value of latter ranges between 0.25 to 1.0 mb, which is much smaller than the cross section itself as a result of threshold effects so only pions with sufficient energy can destroy a  $J/\psi$ .

The rate of  $J/\psi$  production from the hadron gas is given by

$$\frac{dR}{d\tau} = \langle \sigma_{D\bar{D}^*}v \rangle n_D n_{D^*} + \langle \sigma_{D^*\bar{D}}v \rangle n_{D^*}^2 - \langle \sigma_{J/\psi\pi}v \rangle n_{J/\psi} n_\pi, \quad (6)$$

where  $n_D$  and  $n_{D^*}$  are the densities of  $D$  and  $D^*$ , respectively, and their relative value is determined by assuming that they are in chemical equilibrium, i.e.,

$$\frac{n_D}{n_{D^*}} = \frac{1 + (1/3) \exp((m_{D^*} - m_D)/T)}{1 + 3 \exp(-(m_{D^*} - m_D)/T)}. \quad (7)$$

In Eq. (6), we have used the fact that the densities of charm and anti-charm mesons are the same. The density of  $J/\psi$  is denoted by  $n_{J/\psi}$ .

The total number of produced  $J/\psi$  is obtained by multiplying the above rate by the volume of the hadron gas and then integrating over time. We find that at RHIC the number of  $J/\psi$  produced from  $D\bar{D}$  annihilation is only 0.005 and is more than an order-of-magnitude smaller than the number of primary  $J/\psi$ . This is different at LHC as a result of the much larger number of  $D\bar{D}$  present in the hadron gas. In Fig. 3, we show by the long-dashed curve the  $J/\psi$  number calculated with a transverse acceleration  $a = 0.1 \text{ c}^2/\text{fm}$  as a function of time in Au+Au collisions at LHC. It is seen that the  $J/\psi$  number increases with time and at freeze out is comparable to the number of primary  $J/\psi$ , which is indicated by the left arrow. We note that calculations carried out for heavy-ion collisions at SPS energies also show a negligible effect on  $J/\psi$  production from  $D\bar{D}$  annihilation in the hadron gas.

For larger values of  $a$ , the hadron gas expands faster so its temperature drops more appreciably, leading to a reduction of the number of produced  $J/\psi$  as shown in Fig. 3 for  $a = 0.15$  and  $0.20 \text{ c}^2/\text{fm}$ . However, the transverse expansion velocity in these cases reaches the velocity of light after a few fm/c, which seems unreasonable. On the other hand, with smaller values of  $a$ , the final  $J/\psi$  number is increased as shown in Fig. 3 for  $a = 0.05 \text{ c}^2/\text{fm}$ .

If pions are out of chemical equilibrium [25],  $J/\psi$  absorption may be enhanced. Using a pion chemical potential of 130 MeV, the final  $J/\psi$  number is reduced only slightly and is still comparable to the primary one. The lifetime of the phase transition also affects the result. If it is doubled, then the number of produced  $J/\psi$  is reduced to about 2.25, while it is increased to 6.15 if the phase transition is instantaneous. Reducing the value of the temperature at which the phase transition occurs does not change much the number of  $J/\psi$  produced from the hadron gas. Using  $T_c = 150 \text{ MeV}$ , the  $J/\psi$  number is reduced only slightly to 3.53 due to the competing effects of increased  $\langle \sigma_{D\bar{D}^*} \rangle$  and shortened lifetime of the hadron gas with decreasing temperature.

The dependence of  $J/\psi$  production on the cross section  $\sigma_{J/\psi\pi \rightarrow D\bar{D}}$  is more significant. If the cross section is an order of magnitude smaller than the one used here as predicted by the perturbative QCD [6] and the effective Lagrangian [23] calculation, then the number of  $J/\psi$  produced from the hadron gas at LHC is only 0.41 and is much less than that of primary ones. It is thus important to have a better determination of this cross section from both experiments and theoretical models.

The above analysis can be generalized to  $\psi'$  production in ultra-relativistic heavy-ion collisions. The primary  $\psi'$  number is about a factor of 5 smaller than the  $\psi$  number, i.e., about 0.019 and 0.682 at RHIC and LHC, respectively. Since the radius of  $\psi'$  is about twice that of  $\psi$ , its absorption cross section by a pion is about four times larger. Using this cross section and the  $\psi'$  mass, we obtain only 0.002  $\psi'$  from  $D\bar{D}$  annihilation in hadron gas at RHIC, again negligible as in the case of  $J/\psi$ . For heavy-ion collisions at LHC, the number of  $\psi'$  produced from the hadron gas is about 1.15 and is more than the number of primary  $\psi'$ . Therefore, there will not be a suppression of  $\psi'$  production at LHC either.

To summarize, we have estimated the number of  $J/\psi$  produced from the reaction  $D\bar{D} \rightarrow J/\psi\pi$  in the hadron gas formed in heavy-ion collisions at RHIC and LHC using the initial conditions determined from the HIJING parton model. Although this is a negligible effect

at RHIC, it may become important at LHC as a result of the appreciable number of  $D$  and  $\bar{D}$  mesons in the hadron gas. We have found that the number of  $J/\psi$  produced from this reaction at LHC is comparable to that produced from initial primary collisions, which are either absorbed by nucleons or dissociated in the quark-gluon plasma formed in the collisions. We thus conclude that in heavy-ion collisions at LHC one may not see a suppression of  $J/\psi$  production. A similar estimate has been made for  $\psi'$  production, where the number of  $\psi'$  produced from the hadron gas may be even larger than the primary ones. To use the  $J/\psi$  and  $\psi'$  yields as signals for the quark-gluon plasma at LHC therefore requires a good understanding of their production from the hadron gas.

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# FIGURES

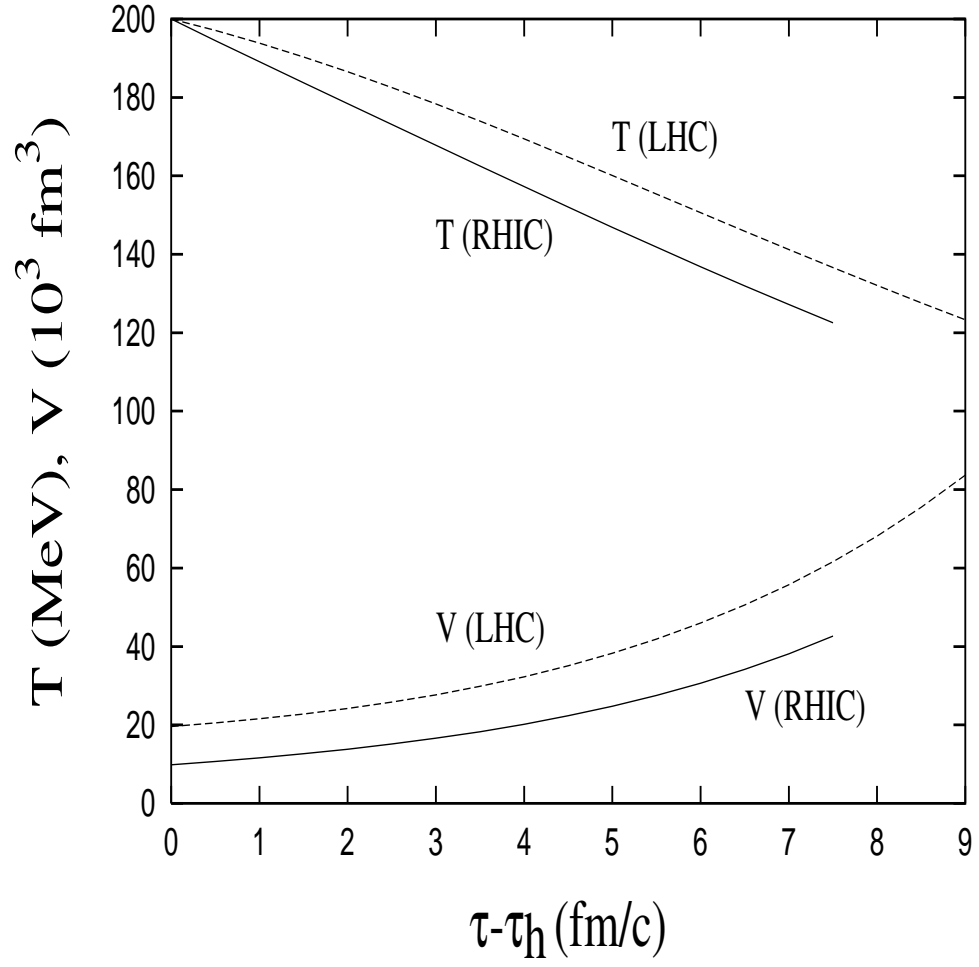


FIG. 1. Time evolution of the temperature and volume of hadron gas in Au+Au collisions at RHIC and LHC.



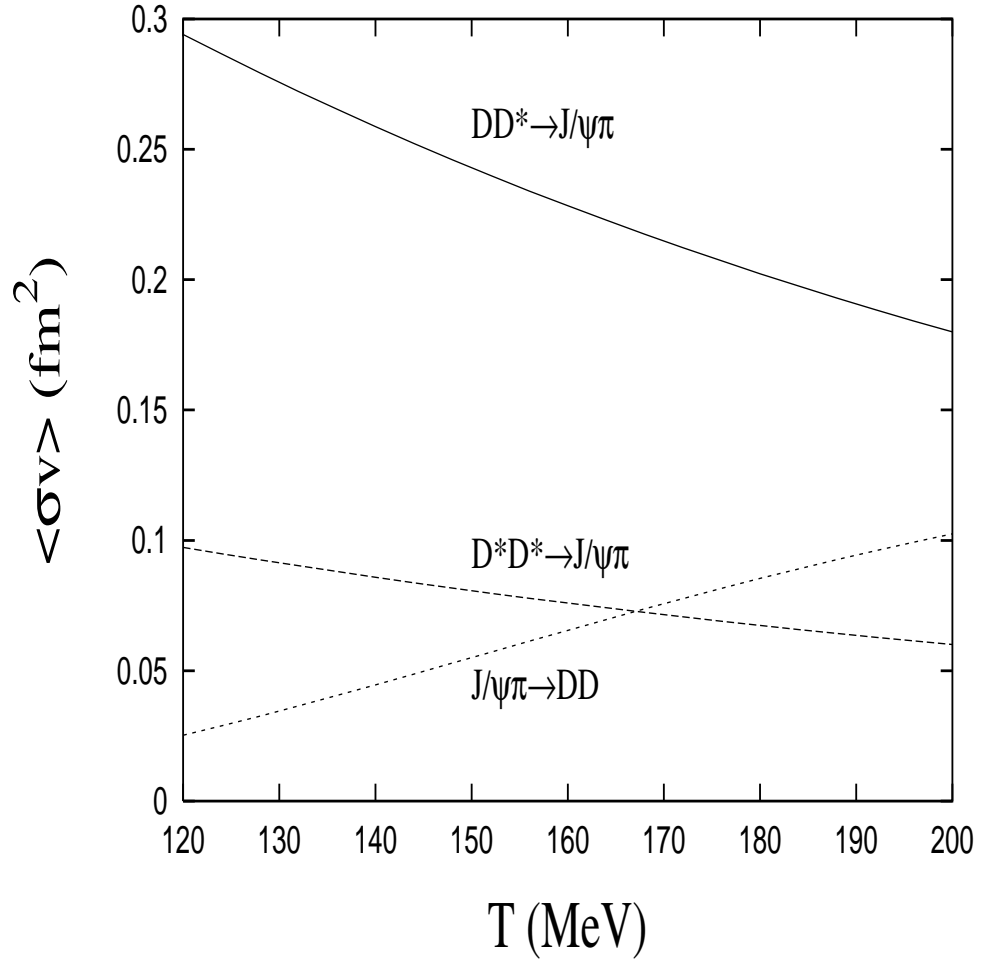


FIG. 2. Temperature dependence of the thermally averaged cross sections:  $\langle \sigma_{DD^*} v \rangle$ ,  $\langle \sigma_{D^*D^*} v \rangle$ , and  $\langle \sigma_{J/\psi \pi} v \rangle$ .

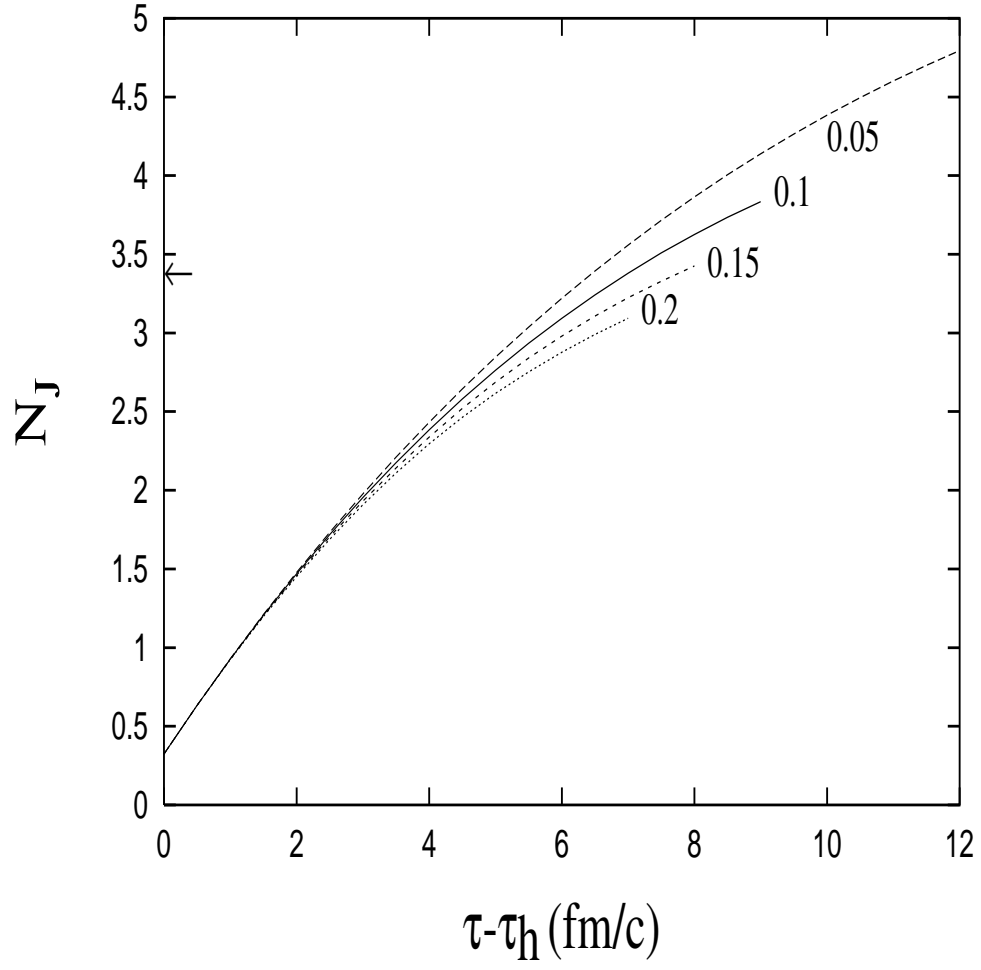


FIG. 3. Time evolution of the abundance of  $J/\psi$  and  $D$  from Au+Au collisions at LHC for different values of transverse acceleration  $a$  ( $\text{c}^2/\text{fm}$ ). The initial  $J/\psi$  number is denoted by the left arrow.